

Programske paradigme - (drugi) primer rezolucije

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Date su nam rečenice:

- The only animals in this house are cats.
- Every animal that loves to gaze at the moon is suitable for a pet.
- When I detest an animal, I avoid it.
- No animals are carnivorous unless they prowl at night.
- No cat fails to kill mice.
- No animals ever like me, except those that are in this house.
- Kangaroos are not suitable for pets.
- None but carnivorous animals kill mice.
- I detest animals that do not like me.
- Animals that prowl at night always love to gaze at the moon.
- Therefore, I always avoid a kangaroo.

Uvedimo formule:

$House(x)$ - x is in this house

$Cat(x)$ - x is a cat

$Gaze(x)$ - x loves to gaze at the moon

$Pet(x)$ - x is suitable for a pet

$Detest(x)$ - I detest x

$Avoid(x)$ - I avoid x

$Carnivorous(x)$ - x is carnivorous

$Prowl(x)$ - x prowls at night

$Kill(x)$ - x kills mice

$Like(x)$ - x likes me

$Kangaroo(x)$ - x is a kangaroo

Transformacijom datih rečenica ovim formulama, dobijamo:

$$\begin{aligned}\forall x(House(x) \Rightarrow Cat(x)) \\ \forall x(Gaze(x) \Rightarrow Pet(x)) \\ \forall x(Detest(x) \Rightarrow Avoid(x)) \\ \forall x(Carnivorous(x) \Rightarrow Prowl(x)) \\ \forall x(Cat(x) \Rightarrow Kill(x)) \\ \forall x(Like(x) \Rightarrow House(x)) \\ \forall x(Kangaroo(x) \Rightarrow \neg Pet(x)) \\ \forall x(Kill(x) \Rightarrow Carnivorous(x)) \\ \forall x(\neg Like(x) \Rightarrow Detest(x)) \\ \forall x(Prowl(x) \Rightarrow Gaze(x))\end{aligned}$$

$$\forall x(Kangaroo(x) \Rightarrow Avoid(x))$$

Šta bi bila negacija zaključka? Važi:

$$\begin{aligned}\neg \forall x(Kangaroo(x) \Rightarrow Avoid(x)) \\ \equiv \exists x \neg (Kangaroo(x) \Rightarrow Avoid(x)) \\ \equiv \exists x \neg (\neg Kangaroo(x) \vee Avoid(x)) \\ \equiv \exists x (Kangaroo(x) \wedge \neg Avoid(x)).\end{aligned}$$

Dakle, za neku konstantu a , važi $Kangaroo(a) \wedge \neg Avoid(a)$, odnosno važi $Kangaroo(a)$ i važi $\neg Avoid(a)$.

Prebacivanjem svih formula u klauzalni oblik¹, dobijamo:

$$\begin{aligned}\forall x(\neg House(x) \vee Cat(x)) \\ \forall x(\neg Gaze(x) \vee Pet(x)) \\ \forall x(\neg Detest(x) \vee Avoid(x)) \\ \forall x(\neg Carnivorous(x) \vee Prowl(x)) \\ \forall x(\neg Cat(x) \vee Kill(x)) \\ \forall x(\neg Like(x) \vee House(x)) \\ \forall x(\neg Kangaroo(x) \vee \neg Pet(x)) \\ \forall x(\neg Kill(x) \vee Carnivorous(x)) \\ \forall x(Like(x) \vee Detest(x)) \\ \forall x(\neg Prowl(x) \vee Gaze(x))\end{aligned}$$

$$\exists x(Kangaroo(x) \wedge \neg Avoid(x))$$

¹Zaključak još uvek sadrži konjunkciju u sebi, ali je u samom dokazu to rešeno razdvajanjem na dva konjunkta (koraci 11 i 12).

Primenimo metod rezolucije:

1. $\neg House(x) \vee Cat(x)$ (premissa)
2. $\neg Gaze(x) \vee Pet(x)$ (premissa)
3. $\neg Detest(x) \vee Avoid(x)$ (premissa)
4. $\neg Carnivorous(x) \vee Prowl(x)$ (premissa)
5. $\neg Cat(x) \vee Kill(x)$ (premissa)
6. $\neg Like(x) \vee House(x)$ (premissa)
7. $\neg Kangaroo(x) \vee \neg Pet(x)$ (premissa)
8. $\neg Kill(x) \vee Carnivorous(x)$ (premissa)
9. $Like(x) \vee Detest(x)$ (premissa)
10. $\neg Prowl(x) \vee Gaze(x)$ (premissa)
11. $Kangaroo(a)$ (negacija zaključka; prvi konjunkt)
12. $\neg Avoid(a)$ (negacija zaključka; drugi konjunkt)
13. $Cat(x) \vee \neg Like(x)$ (rezolucija, 1, 6)
14. $\neg Like(x) \vee Kill(x)$ (rezolucija, 5, 13)
15. $Kill(x) \vee Detest(x)$ (rezolucija, 9, 14)
16. $Detest(x) \vee Carnivorous(x)$ (rezolucija, 8, 15)
17. $Carnivorous(x) \vee Avoid(x)$ (rezolucija, 3, 16)
18. $Avoid(x) \vee Prowl(x)$ (rezolucija, 4, 17)
19. $Avoid(x) \vee Gaze(x)$ (rezolucija, 10, 18)
20. $Avoid(x) \vee Pet(x)$ (rezolucija, 2, 19)
21. $Avoid(x) \vee \neg Kangaroo(x)$ (rezolucija, 7, 20)
22. $\neg Kangaroo(a)$ (unifikacija $\sigma[x \rightarrow a]$, 12, 21)
23. \perp (rezolucija, 11, 22)
24. QED